Cosmological Perturbations in Alternative Cosmological Models

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McGill University

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Outline

Plan

Theory of Cosmological Perturbations: Basics

Cosmological fluctuations connect early universe theories with observations

- Fluctuations of matter → large-scale structure
- Fluctuations of metric → CMB anisotropies
- N.B.: Matter and metric fluctuations are coupled

Key facts:

- 1. Fluctuations are small today on large scales
- → fluctuations were very small in the early universe
- → can use linear perturbation theory
- 2. Sub-Hubble scales: matter fluctuations dominate
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Quantum Theory of Linearized Fluctuations

V. Mukhanov, H. Feldman and R.B., *Phys. Rep. 215:203 (1992)*

Step 1: Metric including fluctuations

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\begin{aligned}
ds^2 &= a^2[(1+2\Phi)d\eta^2 - (1-2\Phi)d\mathbf{x}^2] \\
\varphi &= \varphi_0 + \delta\varphi
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Note: Φ and $\delta\varphi$ related by Einstein constraint equations

Step 2: Expand the action for matter and gravity to second order about the cosmological background:

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Sasaki-Mukhanov variable

M. Sasaki, *Prog. Theor. Phys.* **76**, *1036 (1986)*

where

$$v \sim a\zeta$$

where ζ is the curvature fluctuation in co-moving coordinates.

Step 3: Resulting equation of motion (Fourier space)

$$v_k'' + (k^2 - \frac{z''}{z})v_k = 0$$

Features:

- oscillations on sub-Hubble scales
- squeezing on super-Hubble scales $v_k \sim z$

Quantum vacuum initial conditions:

$$v_k(\eta_i) = (\sqrt{2k})^{-1}$$

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Scale Invariance

Power spectrum:

$$\mathcal{P}_v(k,t) \equiv k^3 |v_k(t)|^2$$

Scale invariance:

$$\mathcal{P}_\zeta(k,t) \sim k^{n-1} \sim k^0 \,.$$

$n = 1$ corresponds to scale-invariance.

Plan

Current Paradigm for Early Universe Cosmology

The Inflationary Universe Scenario is the current paradigm of early universe cosmology.

Successes:

- Solves horizon problem
- Solves flatness problem
- Solves size/entropy problem
- Provides a causal mechanism of generating primordial cosmological perturbations (Chibisov & Mukhanov, 1981).

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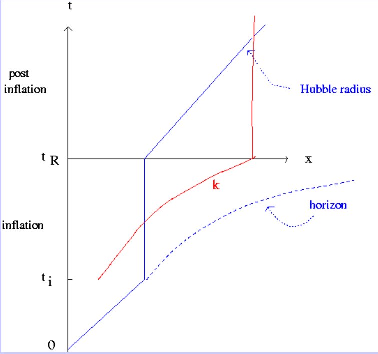
Structure formation in inflationary cosmology



N.B. Perturbations originate as quantum vacuum fluctuations.

Key Features

- Horizon exponentially larger than the Hubble radius.
- Comoving scales probed in current observations originate on sub-Hubble lengths.

Origin of Scale-Invariance

Heuristic analysis [W. Press, 1980, K. Sato,1981]: time-translation symmetry of de Sitter phase → scale-invariance of spectrum.

Mathematical analysis [Mukhanov and Chibisov, 1982]:

$$\begin{aligned}
\mathcal{P}_\zeta(k,t) &\propto \mathcal{P}_v(k,t) \\
&\sim k^3 \Big(\frac{a(t)}{a(t_H(k))}\Big)^2 |v_k(t_H(k))|^2 \\
&\sim k^3 \eta_H(k)^2 |v_k(t_H(k))|^2 \\
&\sim k^0
\end{aligned}$$

using $a(\eta) \sim \eta^{-1}$ in the de Sitter phase and $\eta_H(k) \sim k^{-1}$.

Power of the Sasaki-Mukhanov Variable

- Connecting fluctuations during the period of inflation with those after reheating requires knowledge of matching conditions.
- Naively matching Φ across the reheating surface yields a result which is wrong by 10^{-12}.
- Matching of v gives the right result (conservation of ζ on super-Hubble scales).

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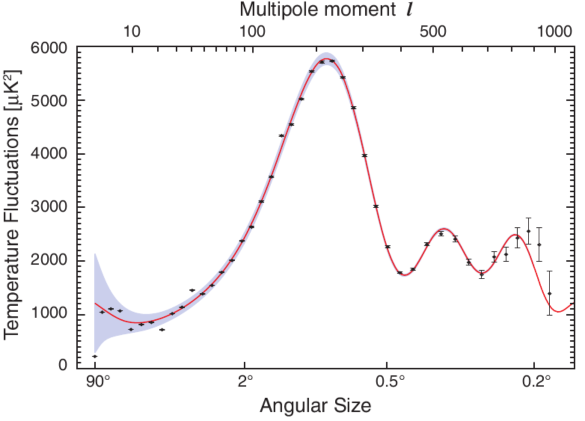
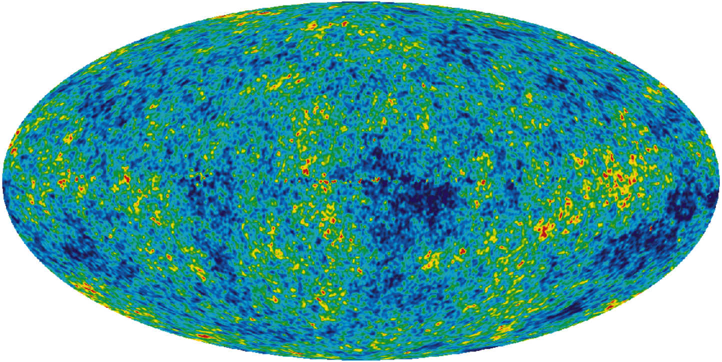
Further Predictions

- Fluctuations adiabatic.
- Fluctuations passive.
- Fluctuations approximately Gaussian.



Credit: NASA/WMAP Science Team



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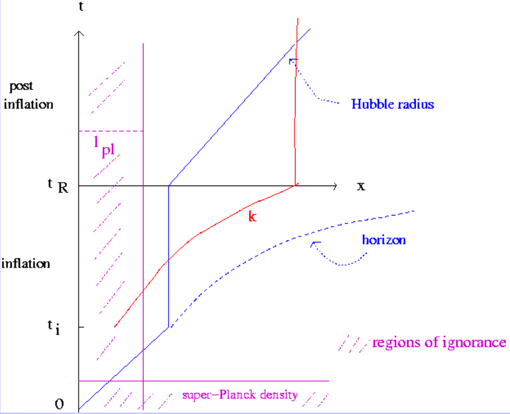
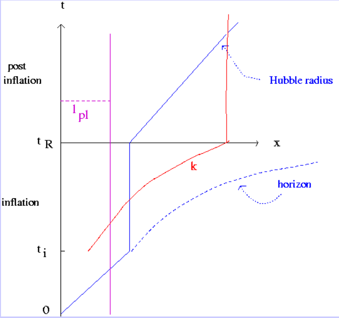
Conceptual Problems of Inflationary Cosmology

- Nature of the scalar field φ (the “inflaton")
- Conditions to obtain inflation (initial conditions, slow-roll conditions, graceful exit and reheating)
- Amplitude problem
- Trans-Planckian problem
- Singularity problem
- Cosmological constant problem
- Applicability of General Relativity

Zones of Ignorance



Trans-Planckian Problem



- **Success of inflation**: At early times scales are inside the Hubble radius → causal generation mechanism is possible.
- **Problem:** If time period of inflation is more than $70H^{-1}$, then $\lambda_p(t) < l_{pl}$ at the beginning of inflation
- → new physics MUST enter into the calculation of the fluctuations.

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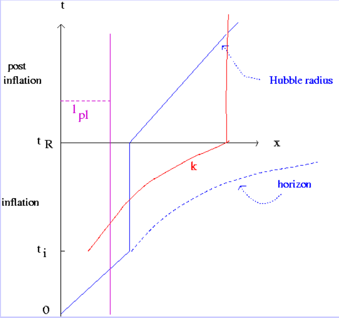

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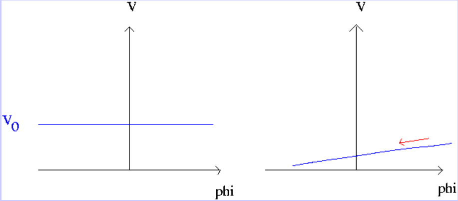
Cosmological Constant Problem



- Quantum vacuum energy does not gravitate.
- Why should the almost constant $V(\varphi)$ gravitate?

$$\frac{V_0}{\Lambda_{obs}} \sim 10^{120}$$

Applicability of GR

- In all approaches to quantum gravity, the Einstein action is only the leading term in a low curvature expansion.
- Correction terms may become dominant at much lower energies than the Planck scale.
- Correction terms will dominate the dynamics at high curvatures.
- The energy scale of inflation models is typically $\eta \sim 10^{16}\mathrm{GeV}$.
- $\rightarrow \eta$ too close to m_{pl} to trust predictions made using GR.

Message

- Current realizations of inflation have conceptual problems.
- This is one of the motivations for studying alternative early universe cosmologies.
- We need new fairy tales!
- Second motivation: in order for inflationary cosmology to make further progress, one needs new alternatives against which inflation can be measured.

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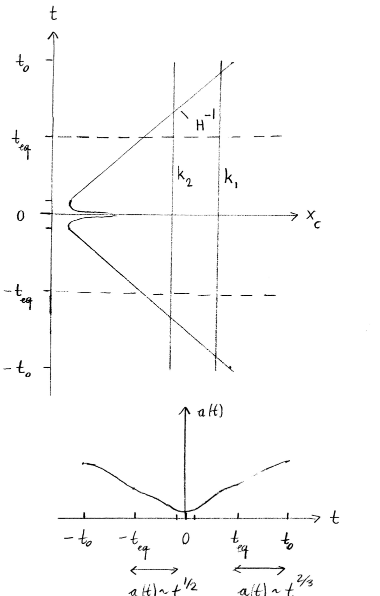
Plan

1. Theory of Cosmological Perturbations
2. Application to Inflationary Cosmology
3. Matter Bounce and Structure Formation
   - Structure Formation
   - Models for a Nonsingular Bounce
   - S-Brane Bounce
4. Ekpyrotic Scenario and Structure Formation
5. String Gas Cosmology and Structure Formation
   - Background
   - Structure Formation
6. Conclusions

Matter Bounce Scenario

F. Finelli and R.B., *Phys. Rev. D65, 103522 (2002)*, D. Wands, *Phys. Rev. D60 (1999)*



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Origin of Scale-Invariant Spectrum

- The initial vacuum spectrum is blue:

$$P_{\zeta}(k) = k^3|\zeta(k)|^2 \sim k^2$$

- The curvature fluctuations grow on super-Hubble scales in the contracting phase:

$$v_k(\eta) = c_1\eta^2 + c_2\eta^{-1} ,$$

- For modes which exit the Hubble radius in the matter phase the resulting spectrum is scale-invariant:

$$\begin{aligned} P_{\zeta}(k, \eta) &\sim k^3|v_k(\eta)|^2 a^{-2}(\eta) \\ &\sim k^3|v_k(\eta_H(k))|^2 \left(\frac{\eta_H(k)}{\eta}\right)^2 \sim k^{3-1-2} \end{aligned}$$

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Transfer of the Spectrum through the Bounce

- In a nonsingular background the fluctuations can be tracked through the bounce explicitly (both numerically in an exact manner and analytically using matching conditions at times when the equation of state changes).
- Explicit computations have been performed in the case of quintom matter (Y. Cai et al, 2008), mirage cosmology (R.B. et al, 2007), Horava-Lifshitz bounce (X. Gao et al, 2010).
- Result: On length scales larger than the duration of the bounce the spectrum of v goes through the bounce unchanged.
- Result: There are models in which the spectrum of curvature fluctuations does not go through the bounce unchanged (S-brane bounce: S. Patil, R.B. et al, to be published).

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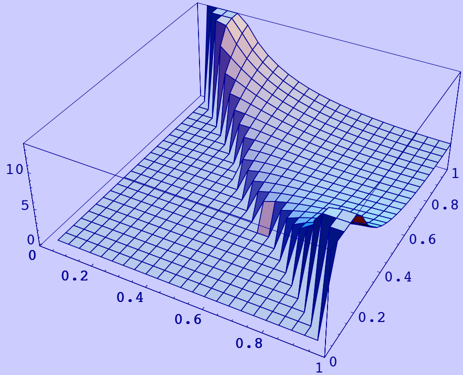
Specific Prediction: Bispectrum of the Matter Bounce Scenario

Y. Cai, W. Xue, R.B. and X. Zhang, *JCAP 0905:011 (2009)*



Overview

In order to obtain a bouncing cosmology it is necessary to:

- either modify the gravitational action
- or introduce a new form of matter which violates the NEC (null energy condition).

It is well motivated to consider models which go beyond the standard coupling of General Relativity to matter obeying the NEC - any approach to quantizing gravity yields terms in the effective action for the metric and matter fields which contain higher derivatives.

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C. Lin, R.B. and L. Perreault Levasseur, 2010

Idea: Instead of a ghost field use a ghost condensate.

Ghost condensate: Take a field ϕ which when expanded about $\phi = 0$ has ghost-like excitations. Construct a Lagrangian such that there is a stable condensate which breaks local Lorentz invariance and about which the model is perturbatively ghost free.

$$\mathcal{L} = M^4 P(X) - V(\phi) \, , \quad X \equiv -g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$$

$$P(X) = \frac{1}{8}(X - c^2)^2 \, ,$$

Ghost condensate for cosmology:

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Ghost Condensate Bounce II

Friedmann Equations for ghost condensate matter:

$$\begin{aligned} 3M_p^2H^2 &= M^4(2XP' - P) + V + \rho_m \, , \\ 2M_p^2\dot{H} &= -2M^4XP' - (1+w_m)\rho_m \, . \end{aligned}$$

Consider fluctuations of the ghost condensate field:

$$\phi(t) = ct + \pi(t) \, .$$

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If $\dot{\pi} < 0$ then the ghost condensate carrier negative gravit. energy.

Equation of motion for the ghost condensate field (leading order in $\dot{\pi}$):

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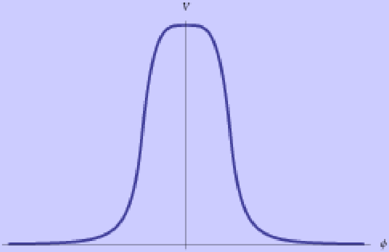

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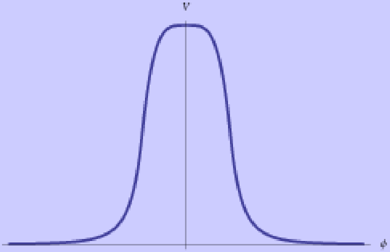
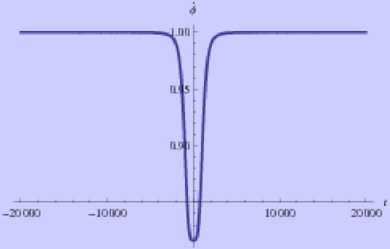


Ghost Condensate Bounce IV

Condition for the bounce point:

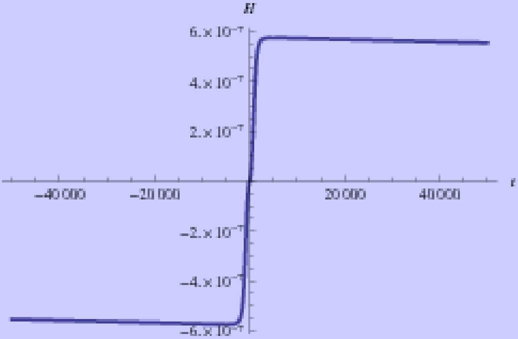
$$M^4 c^3 \dot{\pi} = -V .$$

Numerical results:



Ghost Condensate Bounce V

Numerical results (ctd.):



Fluctuations through the Bounce

C. Lin, R.B., L. P. Levasseur, JCAP **1104**, 019 (2011)

- On length scales larger than the duration of the bounce phase the spectrum of curvature fluctuations is **unchanged** in slope across the bounce.
- The amplitude of the scaler mode will be amplified.
- The spectrum of gravitational waves is scale invariant, but the amplitude does not increase during the bounce.

Hořava-Lifshitz Gravity

P. Hořava, *Phys. Rev.* **D79**, *084008 (2009)*

Power-counting renormalizable quantum theory of gravity in 4d based on anisotropic scaling between space and time:

$$t \rightarrow l^z t \, , \quad x^i \rightarrow l x^i \, .$$

Usual metric degrees of freedom:

$$ds^2 \, = \, -N^2 dt^2 + g_{ij}(dx^i + N^i dt)(dx^j + N^j dt) \, .$$

Most general action consistent with residual symmetries and power-counting renormalizability:

$$S^g \, = \, S^g_K + S^g_V \, .$$

Hořava-Lifshitz Gravity II

Kinetic piece of the action:

$$S_K^g = \frac{2}{\kappa^2} \int dt d^3x \sqrt{g} N \left(K_{ij} K^{ij} - \lambda K^2 \right) ,$$

where

$$K_{ij} = \frac{1}{2N} [\dot{g}_{ij} - \nabla_i N_j - \nabla_j N_i] ,$$

Potential piece of the action (special case - detailed balance):

$$\begin{aligned} S_V^g &= \int dt d^3x \sqrt{g} N [-\frac{\kappa^2}{2w^4} C_{ij} C^{ij} + \frac{\kappa^2 \mu}{2w^2} \epsilon^{ijk} R_{il} \nabla_j R^l_k \\ &- \frac{\kappa^2 \mu^2}{8} R_{ij} R^{ij} + \frac{\kappa^2 \mu^2}{8(1-3\lambda)} \left(\frac{1-4\lambda}{4} R^2 + \Lambda R - 3\Lambda^2 \right)] \end{aligned}$$

Hořava-Lifshitz Bounce I

R.B., *Phys. Rev.* **D80, 043516 (2009)**

In the presence of nonvanishing spatial curvature, the higher spatial derivative terms in the geometrical action act as ghost matter.

The FRWL equations become:

$$\frac{6(3\lambda - 1)}{\kappa^2} H^2 = \rho - \frac{3\kappa^2 \mu^2}{8(3\lambda - 1)} \left(\frac{\bar{k}}{a^2} - \Lambda \right)^2 ,$$

where $\bar{k}$ is the spatial curvature constant;

$$\frac{2(3\lambda - 1)}{\kappa^2} \dot{H} = -\frac{(1+w)\rho}{2} + \frac{\kappa^2 \mu^2}{4(3\lambda - 1)} \left(\frac{\bar{k}}{a^2} - \Lambda \right) \frac{\bar{k}}{a^2} .$$

Hořava-Lifshitz Bounce II

- $\bar{k}/a^4$ term acts as ghost radiation!
- For a general potential there is also ghost anisotropic stress.
- $\rightarrow$ in the presence of spatial curvature a cosmological bounce will occur.
- The bounce is stable against the presence of radiation.
- The bounce is marginally stable against the presence of anisotropic stress.

Fluctuations in the Hořava-Lifshitz Bounce

X. Gao, Y. Wang, W. Xue and R. B., JCAP **1002**:020 (2010)

Result: Spectrum of cosmological perturbations does not change its spectral index during the bounce.

S-Brane Bounce

C.Kounnas, H. Partouche and N. Toumbas, arXiv:1106.0946

- Type II superstring theory compactified on

$$\mathcal{M} \,=\, S^1(R_0) \times T^3 \times \mathcal{F}_6 \,,$$

- Euclidean time radius $R_0 = \beta/(2\pi)$.
- Gravitomagnetic fluxes threading the Euclidean time cycle and cycles of the internal space.
- Leads to T-duality about the Euclidean time cycle (thermal duality)

$$Z(\beta) \,=\, Z(\beta_c^2/\beta)\,.$$

- Large $T^3 \rightarrow$ effective field theory analysis under good control.
- Assumption: weak string coupling.

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Thermal Phases

- Low temperature phase: $\beta \gg \beta_c$

$$\frac{Z}{V} = n^* \sigma_r \beta^{-3} + \mathcal{O}(e^{-\beta/\beta_c}),$$

- Small β phase: $\beta \ll \beta_c$

$$\frac{Z}{V} = n^* \sigma_r \beta_c^{-6} \beta^3 + \mathcal{O}(e^{-\beta_c/\beta}).$$

- Introducing the duality-invariant temperature

$$T = T_c e^{-|\sigma|} \text{ with } e^{\sigma} = \frac{\beta}{\beta_c}.$$

- we obtain $\frac{Z}{V} = n^* \sigma_r T^3 + \mathcal{O}(e^{-T/T_c})$.

Effective Action

- At the critical temperature: thermal winding states become massless.
- enhanced gauge symmetry at $\beta = \beta_c$.
- Enhanced symmetry states enter the effective low energy action for the light degrees of freedom as an S-brane.
- S-brane: space-like topological defect: $\rho = 0$, $p < 0$.
- S-brane mediates violation of Null Energy Condition.
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Matching Conditions

W. Israel, Nuovo Cim. (1966), J-C. Hwang and E. Vishniac, Ap. J. (1991), N. Deruelle and V. Mukhanov, gr-qc/9503050, R. Durrer and F. Vernizzi, hep-ph/0203275

Matching two solutions of Einstein's equations across a brane. The following conditions must be satisfied:

- Induced metric continous
- extrinsic curvature jumps by a value corresponding to the amplitude of the S-brane source.

Matching for Adiabatic Fluctuations

R. Durrer and F. Vernizzi, hep-ph/0203275

- Start in longitudinal gauge.
- Matching surface: identified with a surface of $\bar{\eta} = \mathrm{const.}$

$$\bar{\eta} \equiv \eta + T \, ,$$

- Metric in terms of the new time:

$$\begin{aligned} ds^2 \; &= \; a^2(\bar{\eta}) \big[d\bar{\eta}^2 \big(1 + 2\Phi - 2T' - 2T\mathcal{H} \big) \\ &+ \; dx^i d\bar{\eta} T_{,i} - dx^2 \big(1 - 2\Phi - 2T\mathcal{H} \big) \big] \, . \end{aligned}$$

- Continuity of the induced metric:

$$[\Phi + T\mathcal{H}]|_{\pm} \; = \; 0 \, ,$$

Matching in the General Case

- Two mode functions of the “Bardeen variable":

$$\Phi(k,\eta) = A_-(k)\frac{\mathcal{H}}{a^2}(\eta) + B_-(k)\,.$$

- where

$$\begin{aligned} A_-(k) &\sim k^{-\mu-1}\,, \text{dominant} \\ B_-(k) &\sim k^{\mu-1}\,, \end{aligned}$$

- with

$$\mu = \frac{5+3w}{2(1+3w)}\,.$$

- For a matter-dominated phase $\mu = 5/2$

Matching in the General Case II

- Connection with ζ:

$$\Phi \, = \, \frac{4\pi G}{k^2} \mathcal{B}(\eta) \zeta' \, ,$$

- Hence, the A-mode in the contracting phase yields a scale-invariant power spectrum for ζ.
- In the expanding phase:

$$\Phi(k, \eta) \, = A_+(k) \frac{\mathcal{H}}{a^2}(\eta) + B_+(k) \, .$$

- B-mode is dominant.

Key Question: How do the two modes couple across the bounce?

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- Case A: Matching across constant energy density hypersurface:
- ζ conserved, trivial mixing between A and B modes.
- $\rightarrow$ scale-invariant spectrum for B after the bounce.
- Case B: S-brane is located on a constant temperature hypersurface, not constant energy density hypersurface:
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Instability against anisotropic stress

$$\rho_{\text{anis}} \sim a^{-6}$$

Anisotropic stress will dominate near the bounce and destroy the quasi-homogeneous bounce (BKL instability).

"Solution" (Y. Cai, R. B. and D. Easson, 2012)

- Introduce new scalar field with Ekpyrotic potential.
- Arrange that this scalar field comes to dominate after the onset of the radiation phase of contraction, at time $-t_E$.
- For $t > -t_E$ the relative impact of anisotropies decreases since $\rho(\varphi_E) \sim a^{-p}$ with $p > 6$.

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Plan

Setup

- Space-time given by General Relativity
- Matter given by a scalar field φ with negative exponential potential.
- Yields contracting phase with $w >> 1$
- $\rho_{\varphi} \sim a^{-q}$ with $q > 6$
- $\rightarrow$ no BKL instability.
- Motivated by Hořava-Witten theory: 11-d supergravity model with time-dependent radius r of an orbifold direction: $\varphi \sim \ln r$.

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Since the contraction is slow, scales exit the Hubble radius at very similar values of the scale factor. Hence, the vacuum spectrum of v is preserved on super-Hubble scales (D. Lyth (2001), R.B. and F. Finelli (2001), J-C. Hwang (2001))

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Matching Conditions

R. Durrer and F. Vernizzi, hep-ph/0203275

- If matching occurs on constant energy density surfaces, then the vacuum spectrum of ζ is preserved.
- If the matching occurs on a surface which differs from the constant energy density hypersurface on IR scales, then the final spectrum of Φ is scale-invariant.
- This occurs if the higher-dimensional origin of the Ekpyrotic model is taken into account (T. Battefeld, R.B. and S. Patil, 2004).
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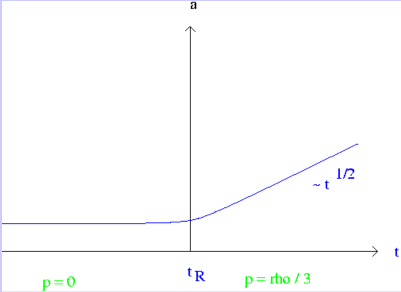
Further Predictions

- Fluctuations adiabatic.
- Fluctuations passive.
- Fluctuations approximately Gaussian.

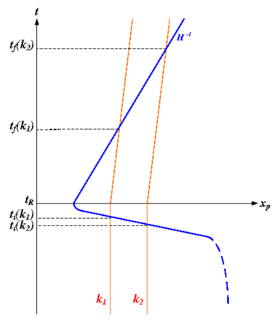
Plan

Dynamics

We consider the following background dynamics for the scale factor $a(t)$:



Space-time sketch



Features

- No horizon problem [horizon $\neq$ Hubble radius]
- Flatness problem mitigated
- No structure formation problem
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- Entropy (size) problem not solved

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Principles of String Gas Cosmology

R.B. and C. Vafa, *Nucl. Phys. B316:391 (1989)*

Idea: make use of the new symmetries and new degrees of freedom which string theory provides to construct a new theory of the very early universe.

Assumption: Matter is a gas of fundamental strings

Assumption: Space is compact, e.g. a torus.

Key points:

- New degrees of freedom: string oscillatory modes
- Leads to a maximal temperature for a gas of strings, the Hagedorn temperature
- New degrees of freedom: string winding modes
- Leads to a new symmetry: physics at large R is equivalent to physics at small R

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T-Duality

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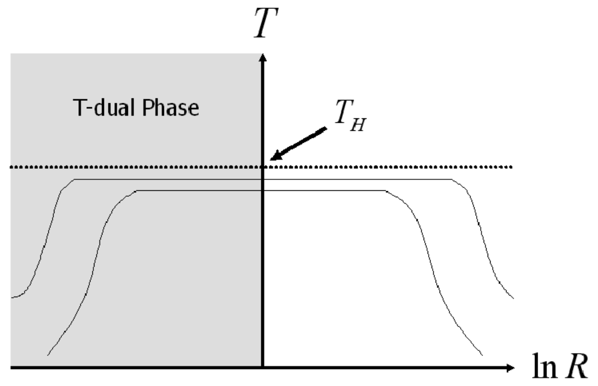
- Momentum modes: $E_n = n/R$
- Winding modes: $E_m = mR$
- Duality: $R \rightarrow 1/R \quad (n, m) \rightarrow (m, n)$
- Mass spectrum of string states unchanged
- Symmetry of vertex operators
- Symmetry at non-perturbative level → existence of D-branes

Adiabatic Considerations

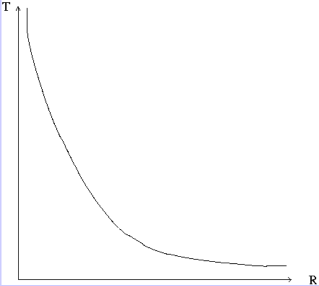
R.B. and C. Vafa, *Nucl. Phys. B316:391 (1989)*

Temperature-size relation in string gas cosmology



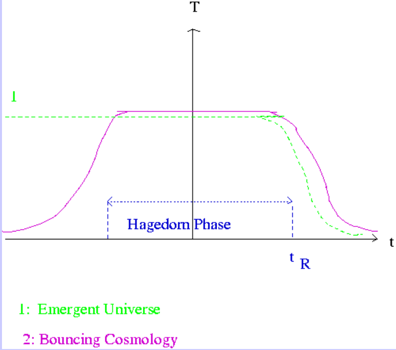
Singularity Problem in Standard and Inflationary Cosmology

Temperature-size relation in standard cosmology



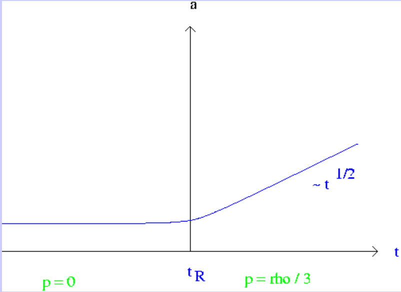
Dynamics

Assume some action gives us $R(t)$



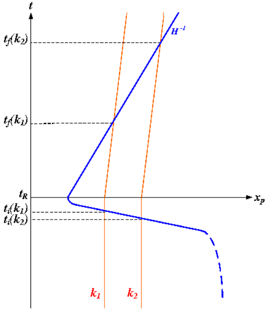
Dynamics II

We will thus consider the following background dynamics for the scale factor $a(t)$:



Structure formation in string gas cosmology

A. Nayeri, R.B. and C. Vafa, *Phys. Rev. Lett. 97:021302 (2006)*



N.B. Perturbations originate as thermal string gas fluctuations.

Method

- Calculate matter correlation functions in the Hagedorn phase (neglecting the metric fluctuations)
- For fixed k, convert the matter fluctuations to metric fluctuations at Hubble radius crossing $t = t_i(k)$
- Evolve the metric fluctuations for $t > t_i(k)$ using the usual theory of cosmological perturbations

Extracting the Metric Fluctuations

Ansatz for the metric including cosmological perturbations and gravitational waves:

$$ds^2 = a^2(\eta)\big((1+2\Phi)d\eta^2 - [(1-2\Phi)\delta_{ij} + h_{ij}]dx^i dx^j\big) .$$

Inserting into the perturbed Einstein equations yields

$$\langle|\Phi(k)|^2\rangle = 16\pi^2 G^2 k^{-4} \langle\delta T^0{}_0(k)\delta T^0{}_0(k)\rangle ,$$

$$\langle|h(k)|^2\rangle = 16\pi^2 G^2 k^{-4} \langle\delta T^i{}_j(k)\delta T^i{}_j(k)\rangle .$$

Power Spectrum of Cosmological Perturbations

Key ingredient: For thermal fluctuations:

$$\langle \delta\rho^2 \rangle = \frac{T^2}{R^6} C_V .$$

Key ingredient: For string thermodynamics in a compact space

$$C_V \approx 2 \frac{R^2/\ell_s^3}{T(1 - T/T_H)} .$$

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Power spectrum of cosmological fluctuations

$$\begin{aligned} P_{\Phi}(k) &= 8G^2 k^{-1} < |\delta\rho(k)|^2 > \\ &= 8G^2 k^2 < (\delta M)^2 >_R \\ &= 8G^2 k^{-4} < (\delta\rho)^2 >_R \\ &= 8G^2 \frac{T}{\ell_s^3} \frac{1}{1 - T/T_H} \end{aligned}$$

Key features:

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Comments

- Evolution for $t > t_i(k)$: $\Phi \simeq \mathrm{const}$ since the equation of state parameter $1 + w$ stays the same order of magnitude unlike in inflationary cosmology.
- Squeezing of the fluctuation modes takes place on super-Hubble scales like in inflationary cosmology → acoustic oscillations in the CMB angular power spectrum
- In a dilaton gravity background the dilaton fluctuations dominate → different spectrum [R.B. et al, 2006; Kaloper, Kofman, Linde and Mukhanov, 2006]

Further Predictions

- Fluctuations adiabatic.
- Fluctuations passive.
- Fluctuations approximately Gaussian (by central limit theorem - thermal scale is much smaller than scales which are probed in current experiments - B. Chen, Y. Wang, W. Xue and R.B., 2007)
- Non-Gaussianities may exist in the form of a scaling network of stable cosmic superstrings.

Spectrum of Gravitational Waves

R.B., A. Nayeri, S. Patil and C. Vafa, *Phys. Rev. Lett. (2007)*

$$\begin{aligned} P_h(k) &= 16\pi^2 G^2 k^{-1} < |T_{ij}(k)|^2 > \\ &= 16\pi^2 G^2 k^{-4} < |T_{ij}(R)|^2 > \\ &\sim 16\pi^2 G^2 \frac{T}{\ell_s^3}(1 - T/T_H) \end{aligned}$$

Key ingredient for string thermodynamics

$$< |T_{ij}(R)|^2 > \sim \frac{T}{l_s^3 R^4}(1 - T/T_H)$$

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- $C_V(R) \sim R^2$ obtained from a thermal gas of strings provided there are winding modes which dominate.
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Note: Specific higher derivative toy model: T. Biswas, R.B., A. Mazumdar and W. Siegel, 2006

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Plan

Conclusions

- Theory of cosmological perturbations is well established and is applicable in any background cosmology.
- Mild assumption: gravitational theory reduces to GR in the infrared.
- The use of the Sasaki-Mukhanov variable simplifies the analysis and clarifies the physics.
- Applications to the matter bounce scenario, to the Ekpyrotic scenario and to string gas cosmology have been worked out.
- Matching conditions across a non-singular bounce can be non-trivial.